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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2022

MTMGDSE03T-MATHEMATICS (DSE2)

NUMERICAL METHODS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Write down the relations of Central difference operator, δ and Average operator, μ with the shift operator E.

(b) Obtain two consecutive integers between which there is a root of $x^3+x+5=0$.

(c) Write down the number $\frac{2}{3}$ correct upto 5 significant figures and find relative error.

(d) Why is the Newton-Raphson method for computing a simple root of an equation $f(x)=0$ called method of tangents?

(e) Construct a linear interpolation for $f(x)$ with $f(1)=3$ and $f(2)=-5$.

(f) Show that $\Delta\log f(x)=\log[1+\Delta f(x)/f(x)]$

(g) Find the value of $f'(0.2)$ using the table of values of $f(x)$

x	0.2	0.4	0.6
$f(x)$	1.6596	1.6698	1.6804

(h) Using trapezoidal rule compute $\int_0^2 f(x)\,dx$. Given

x	0	1	2
$f(x)$	1.6	3.8	8.2

2. (a) Find a real root of the equation $3x-\cos x-1=0$ correct to two significant figures by using Newton Raphson method. 4

(b) Discuss method of bisection for computing a real root of an equation $f(x)=0$. 4

3. (a) Find Lagrange's interpolation polynomial for the function $f(x)=\sin\pi x$, when $x_0=0$, $x_1=\frac{1}{6}$, $x_2=\frac{1}{2}$. Also compute the value of $\sin\frac{\pi}{3}$ and estimate the error. 3+1+1

(b) Find $f(5)$, given that $f(0)=-2$, $f(1)=4$, $f(2)=6$, $f(3)=10$ and third difference being constant. 3

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4. (a) Solve the equation 6

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8$$

by the method of matrix factorization

(b) Round off the number 40.3586 and 0.0056812 to four significant digits. 2

5. (a) Find the missing terms in the following table: 4

x	0	1	2	3	4	5
y	1	5	–	121	–	781

(b) Use of Stirling interpolation formula prove that 4

$$\frac{d}{dx}f(x)=\frac{2}{3}[f(x+1)-f(x-1)]-\frac{1}{12}[f(x+2)-f(x-2)],$$

considering the differences upto third order.

6. (a) Compute $f(0.5)$ from the following table 4

x	0	1	2	3
$f(x)$	1	2	11	34

(b) Show that nth order difference of a polynomial of degree n are constant. Does the converse of the result true? 4

7. (a) Evaluate numerically the integration $\int_0^1 \frac{1}{1+x}dx$, by Simpson's $\frac{1}{3}$rd rule taking 6 equal subintervals. 4

(b) If $f(x)$ is a polynomial of degree 2, prove that 4

$$\int_0^1 f(x)dx=[5f(0)+8f(1)-f(2)]\Big/12\,.$$

8. (a) Compute by the method of fixed point iteration method the positive root of the equation $x^2-x-0.1=0$ correct upto three significant figures. 4

(b) Find the real root of the equation $x^3-x-1=0$ by Regula-Falsi method correct upto two significant figures. 4

9. (a) Use Euler's method with $h=0.2$ to find the solution of $\frac{dy}{dx}=2x+y$, $y(0)=1$ at $x=0.4$. 5

(b) Find the location of the positive roots of $x^3-9x+1=0$, and evaluate the smallest one by bisection method correct to two decimal places. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2022

MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours　　　　　Full Marks: 50

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GROUP-A

Full Marks-10

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Is the set $X=\{(x,y): x^2+y^2\le 4\}$ is convex? Justify your answer.

(b) In the following equations find the basic solution with x_3 as the non-basic variable

$$x_1+4x_2-x_3=3$$
$$5x_1+2x_2+3x_3=4$$

(c) Find a basic feasible solution of the equations $x_1+x_2+x_3=8$, $3x_1+2x_2=18$

(d) Find the extreme points, if any, of the set $S=\{(x,y): 2x+3y=6\}$

(e) Draw the convex hull of the points $(0,0)$, $(0,1)$, $(1,2)$, $(1,1)$, $(4,0)$.

(f) Write down the dual of the following L.P.P.:

Maximize $Z=3x_1+5x_2$

Subject to $x_1+2x_2\le 5$

$x_1-x_2=7$

$x_1,\ x_2\ge 0$

(g) Determine the position of the point $(-1, 2, 5, 3)$ relative to the hyperplane

$$4x_1+6x_2+x_3-3x_4=4$$

(h) Find the number of basic feasible solutions of the following L.P.P.:

Maximize $Z=2x_1+3x_2$

Subject to $x_1+x_2\ge 2$

$x_1-x_2\le 1$

$x_1,\ x_2\ge 0$

(i) What is the criterion for no feasible solution in two-phase method?

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GROUP-B

Full Marks-40

Answer any *five* questions from the following $8\times 5=40$

2. (a) Solve the following L.P.P using graphical method 4

Maximize $Z=2x_1+x_2$

Subject to $4x_1+3x_2\le 12$

$4x_1+x_2\le 8$

$4x_1-x_2\le 8$

$x_1,\ x_2\ge 0$

(b) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food Y contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

3. (a) Use Simplex method to solve the L.P.P. 4

Maximize $Z=x_1+2x_2+4x_3$

Subject to $3x_1+5x_2+2x_3\le 6$

$4x_1+4x_3\le 7$

$2x_1+4x_2-x_3\le 10$

$x_1,x_2,x_3\ge 0$

(b) Show that the vectors $(1,-2,0),(3,1,2),(5,-1,4)$ form a basis in E^3. 4

4. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 5

(b) Find a supporting hyperplane of the convex set 3

$S=\{(x,y): x+2y\le 4, 3x+y\le 6, x\ge 0, y\ge 0\}$

5. (a) $x_1=1,\ x_2=1,\ x_3=1,\ x_4=0$ is a feasible solution of the system of equations 5

$x_1+2x_2+4x_3+x_4=7$

$2x_1-x_2+3x_3-2x_4=4$

Reduce the feasible solution to two different basic feasible solutions.

(b) Prove that a hyperplane is a convex set. 3

6. (a) Obtain a basic feasible solution of the following L.P.P. from the feasible solution (2, 3, 1) 4

Maximize $Z=x_1+2x_2+4x_3$

Subject to $2x_1+x_2+4x_3=11$

$3x_1+x_2+5x_3=14$

$x_1,\ x_2,x_3\ge 0$

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(b) Prove that the intersection of two convex sets is also a convex set. 4

7. (a) Solve by Charnes Big M-method the following L.P.P. 6

Maximize $Z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1,\ x_2 \geq 0$

(b) Discuss whether the set of points $(0, 0), (0, 1), (1, 0), (1, 1)$ on the xy-plane is a convex set or not. 2

8. (a) Prove that dual of a dual is a primal. 4

(b) Obtain the dual problem of the following L.P.P. 4

Maximize $Z = -x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \leq 5$

$x_1 + 6x_2 \leq 9$

$x_1,\ x_2,\ x_3 \geq 0$

9. (a) Find the points which generate the convex polyhedron 3

$$S = \{(x_1, x_2) \in E^2 : x_1 + 2x_2 \leq 4,\ x_1 - 2x_2 \leq 2,\ x_1 \geq 0,\ x_2 \geq 0\}$$

(b) Use two-phase method to solve the following L.P.P. 5

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

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NUMERICAL METHODS

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Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Construct a linear interpolation for $f(x)$ with $f(1)=3$ and $f(2)=-5$.

(b) Compute $\int_0^4 2x^3\,dx$, by Simpson's $\frac{1}{3}$ formula and comment on the result.

(c) Derive an iteration formula for computing $\sqrt[3]{a}$, using Newton Raphson method.

(d) What is the condition of convergency of Gauss-Jacobi iteration to solve the system of n linear equations? Is this condition both necessary and sufficient?

(e) Show that the equation $x^2+\ln x=0$ has exactly one root in the interval $\left[\frac{1}{3}, 1\right]$.

(f) If 0.667 be an approximate value of $\frac{2}{3}$, find the percentage error.

(g) What do you mean by Numerical Differentiation?

(h) Show that $\Delta^2 \cos 2x = 4\cos 2x$ where interval of differencing is $\frac{\pi}{2}$.

(i) Define the terms absolute and relative errors.

2. Explain the Newton-Raphson method for computing a simple real root of an equation $f(x)=0$. When does the method fail? Can we apply this method to the equation $x^2-x+\frac{1}{4}=0$? Justify your answer. 4+1+1+2

3. (a) In order to find the root of $x^3-x-1=0$, near $x=1$ which of the following iteration functions give convergent sequences: 2+2

(i) $x=\frac{x+1}{x^2}$ (ii) $x=\sqrt{\frac{x+1}{x}}$

(b) Apply the method of bisection to find a real root up to two significant digits of the equation $x^3-3x-5=0$. 4

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4. (a) Use Lagrange's interpolation to find the value of $f(x)$ for $x = 0.4$ using the table. 5

x	0.3	0.5	0.6
$f(x)$	0.61	0.69	0.72

(b) Find $\Delta^4 f(x)$, where $f(x) = (3x+2)(x-2)(x+1)(5x-1)$ and the interval of differencing is unity. 3

5. What is interpolation? Deduce Newton's forward difference interpolation formula without error term. 8

6. (a) Given the following table: 2+3

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f'(20)$.

(b) If $f(x) = ax$, show that $(E + E^{-1})f(x) = 2f(x)$. 3

7. (a) Calculate $\int_1^2 (x + \frac{1}{x})\,dx$ up to four significant figures by Simpson's $\frac{1}{3}$ rule taking 4 intervals. 4

(b) Obtain trapezoidal rule for numerical integration without the error term. 4

8. Solve the system of equations by LU decomposition method: 8

$$3x + 4y + 2z = 15 \;,\; 5x + 2y + z = 18 \;,\; 2x + 3y + 2z = 10$$

9. Deduce Lagrange's interpolation formula and also prove that Lagrangian functions are invariant under linear transformation. 8

10.(a) For any positive integer k, show that 4

$$\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i}$$

∇ being the backward difference operator.

(b) What do you mean by 'round off' errors in numerical data? Show how these errors are propagated in a difference table. 4

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WEST BENGAL STATE UNIVERSITY

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MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

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Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the extreme points of the following convex set $S=\{(x,\ y):\ y^2 \le 4x\}$.

(b) Find a supporting hyperplane of the convex set

$$S=\{(x,\ y) : x+2y\le 4,\quad 3x+y\le 6,\quad x\ge 0,\quad y\ge 0\}$$

(c) In the following equations find the basic solution with x_3 as the non-basic variable.

$$x_1+4x_2-x_3=3$$
$$5x_1+2x_2+3x_3=4$$

(d) Is $(2\ ,\ 0)$ a feasible solution of the following LPP?

Maximize $Z=x_1+3x_2$

Subject to $3x_1+6x_2\le 8$

$5x_1+2x_2\le 10$

$x_1,\ x_2\ge 0$

(e) Write the following problem in a standard form:

Maximize $Z=x_1+x_2+x_3$

Subject to $|x_1-x_2+x_3|\le 2$

$x_1-x_2-x_3=3$

$x_1,\ x_2,\ x_3\ge 0$

(f) When a basic solution is said to be degenerated?

(g) A hyperplane is given by $x+3y+2z=9$. In which half spaces the points $(1,\,2,\,4)$ and $(-3\ ,1\ ,-5)$ lie?

(h) Give an example of a non-convex set. Explain why it is non-convex.

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(i) Find the number of basic feasible solution of the following LPP:

Maximize $Z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \geq 2$

$x_1 - x_2 \leq 1$

$x_1,\ x_2 \geq 0$

2. (a) Find all basic feasible solution of the following system of equations. 4

$$x_1 + x_2 - x_3 = 2,$$
$$2x_1 + x_2 - x_3 = 3$$

(b) Show that $(1, 2, 1)$ is a feasible solution of the system of equations. 4

$$x_1 - x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$x_1,\ x_2,\ x_3 \geq 0$$

Reduce the feasible solution to a basic feasible solution.

3. (a) Solve: 4

Maximize $Z = 2x_1 - 3x_2$

Subject to $x_1 + x_2 \leq 2$

$2x_1 + 2x_2 \geq 8$

$x_1,\ x_2 \geq 0$

(b) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food Y contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

4. (a) If the feasible region of a linear programming problem is strictly bounded and contains a finite number of extreme points then prove that the objective function of the linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 4

(b) Show that the feasible solution $x_1 = 1,\ x_2 = 1,\ x_3 = 0,\ x_4 = 2$ to the system 4

$$x_1 + x_2 + x_3 = 2$$
$$x_1 + x_2 - 3x_3 = 2$$
$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

is not basic.

5. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 5

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(b) Give an example of 3

(i) Convex hulls in E^2 and E^3

(ii) Convex polyhedron in E^2

(iii) Simplex in zero and one dimension.

6. (a) Prove that in a linear programming problem the optimal hyperplane is a supporting hyperplane to the convex set of feasible solution. 4

(b) Find a supporting hyperplane passing through $(7,-1)$ of the convex set $X=\{(x_1, x_2) : x_1^2+x_2^2 \le 25\}$. 4

7. Solve the following L.P.P. using duality theory. 8

Maximize $Z=4x_1+3x_2$

Subject to $x_1 \le 6$

$x_2 \le 8$

$x_1+x_2 \le 7$

$3x_1+x_2 \le 15$

$-x_2 \le 1$

$x_1, x_2 \ge 0$

8. (a) Solve by Charnes Big M-method the following L.P.P. 6

Minimize $Z=4x_1+2x_2$

Subject to $3x_1+x_2 \ge 27$

$x_1+x_2 \ge 21$

$x_1+2x_2 \ge 30$

$x_1, x_2 \ge 0$

(b) Discuss whether the set of points $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$ on the xy-plane is a convex set or not. 2

9. (a) Given the L.P.P. 5

Maximize $Z=2x_1+3x_2+4x_3$

Subject to $x_1-5x_2+3x_3=7$

$2x_1-5x_2 \le 3$

$3x_2-x_3 \ge 5$

$x_1, x_2 \ge 0$

x_3 is unrestricted in sign. Formulate the dual of the L.P.P.

(b) Prove that if any variable of the primal problem be unrestricted in sign, then the corresponding constraint of the dual will be equality. 3

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10.(a) If for a basic feasible solution x_B of a linear programming problem maximize $z = cx$, subject to $Ax = b$ and $x \geq 0$, we have $z_j - c_j \geq 0$ for every column a_j of A, then prove that x_B is an optimal solution. 5

(b) Check whether $x = 5,\ y = 0,\ z = -1$ is a basic solution of the system of equations. 3

$$x + 2y + z = 4,$$

$$2x + y + 5z = 5$$

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